医学部専門予備校　クエスト　解答速報

日本大学（医）二次試験　数学　　試験日 2 月 11 日（日）

[1] (1) $\displaystyle\int_0^{\frac{2}{5}} (5x-3)^6\,dx$

$$= \left[\frac{1}{35}(5x-3)^7\right]_0^{\frac{2}{5}} = \frac{1}{35}(-1)^7 - \frac{1}{35}(-3)^7 = \underline{\frac{2186}{35}}$$

(2) $\displaystyle\int_0^{\frac{\pi}{12}} \sin\left(2x+\frac{\pi}{3}\right)dx = \left[-\frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{12}}$

$$= -\frac{1}{2}\cos\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{3} = \underline{\frac{1}{4}}$$

(3) $\displaystyle\int_1^{e} \log 2x\,dx = \left[\frac{1}{2}(2x\log 2x - 2x)\right]_1^{e}$

$$= (e\log 2e - e) - (\log 2 - 1) = \underline{(e-1)\log 2 + 1}$$

(4) $\displaystyle\int_1^{2} \frac{2x+3}{x^2+3x}\,dx = \left[\log(x^2+3x)\right]_1^{2}$

$$= \log 10 - \log 4 = \underline{\log\frac{5}{2}}$$

［2］ $f(x)=(1+\cos x)^3-1-3\cos x$

(1) $f'(x)=3(1+\cos x)^2\cdot(-\sin x)+3\sin x$

$=3\sin x\{1-(1+\cos x)^2\}$

$=3\sin x(-2\cos x-\cos^2 x)$

$=\underline{-3\sin x\cos x(\cos x+2)}$

(2)

x	0	$\cdots$	$\frac{\pi}{2}$	$\cdots$	π
$f'(x)$	0	$-$	0	$+$	0
$f(x)$	4	$\searrow$	0	$\nearrow$	2

増減表より、 $f(x)\geqq 0\ (0\leqq x\leqq\pi)$

[3] (1) $\displaystyle\int_0^{\frac{\pi}{2}} t\cos^2 t\,dt = \int_0^{\frac{\pi}{2}} t\cdot\frac{1+\cos 2t}{2}\,dt$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} t\,dt + \frac{1}{2}\int_0^{\frac{\pi}{2}} t\cos 2t\,dt$$

$$= \frac{1}{4}\left[t^2\right]_0^{\frac{\pi}{2}} + \frac{1}{2}\left(\left[\frac{1}{2}t\sin 2t\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 2t\,dt\right)$$

$$= \frac{\pi^2}{16} + \frac{1}{8}\left[\cos 2t\right]_0^{\frac{\pi}{2}} = \underline{\frac{\pi^2-4}{16}}$$

(2) $\displaystyle\int_0^{\frac{\pi}{2}} t\sin t\cos t\,dt = \frac{1}{2}\int_0^{\frac{\pi}{2}} t\sin 2t\,dt$

$$= \left[-\frac{1}{4}t\cos 2t\right]_0^{\frac{\pi}{2}} + \frac{1}{4}\int_0^{\frac{\pi}{2}}\cos 2t\,dt$$

$$= \frac{\pi}{8} + \frac{1}{8}\left[\sin 2t\right]_0^{\frac{\pi}{2}} = \underline{\frac{\pi}{8}}$$

(3) $\displaystyle f(x) = \int_0^{\frac{\pi}{2}} t(\cos t - x\sin t)^2\,dt$

$$= \int_0^{\frac{\pi}{2}}(t\cos^2 t - 2t\sin t\cos t\,x + t\sin^2 t\,x^2)\,dt$$

$$= x^2\int_0^{\frac{\pi}{2}} t\sin^2 t\,dt - 2x\int_0^{\frac{\pi}{2}} t\sin t\cos t\,dt + \int_0^{\frac{\pi}{2}} t\cos^2 t\,dt$$

（(1)と同様に、$\displaystyle\int_0^{\frac{\pi}{2}} t\sin^2 t\,dt = \frac{\pi^2+4}{16}$）

$$= \underline{\frac{\pi^2+4}{16}x^2 - \frac{\pi}{4}x + \frac{\pi^2-4}{16}}$$

$$= \frac{\pi^2+4}{16}\left(x - \frac{2\pi}{\pi^2+4}\right)^2 + \frac{\pi^4-4\pi^2-16}{16(\pi^2+4)}$$

したがって、$\displaystyle f\left(\frac{2\pi}{\pi^2+4}\right) = \underline{\frac{\pi^4-4\pi^2-16}{16(\pi^2+4)}}$（最小値）

<講評>

一次試験を突破した受験生にとってはあまりにも簡単な試験である。ほとんど差がつかないだろう。